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Test for divisibility of a number by 7,13, 23

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Abstract - A number of papers have been published in different journals for knowing, without performing actual division, whether a given number is exactly divisible by 7,13,23, having zero remainder, or the number is not exactly divisible by 7,13,23 respectively. All the methods published are bit complicated and requires long calculations, specially, for the numbers having many numbers of digits. The author is proposing a much simpler method, which is quicker, faster and can be evaluated without using pen and paper. During testing a number by 23, some calculations can be done orally with little difficulty. A common man, who knows simple addition, subtraction, multiplication, division can use these methods. The method is applicable to a number having any number of digits.

Index Terms – calculation, divisibility, division, numbers, number theory, seven, thirteen, twenty-three

1. INTRODUCTION

Some methods, for testing, whether a given number is exactly divisible by 7 or not, are listed below: -

A. Methodology for computation of numerical value of numbers. In this method digits of the given number is multiplied by 1,3,2,6,4,5,1,3,2,6,4,5………so on, respectively starting from the digit placed at unit place .All values are added. If the sum is divisible by 7 the given number is exactly divisible by 7.
B. A divisibility rule called 6-9 method. In this method, unit place digit is multiplied by 9 and remaining number is multiplied by 6. If the sum is divisible by 7, the given number is divisible by 7. This method has been published in your journal Vol-4, Issue-4, April 2013.
C. A divisibility rule in which unit place digit is doubled and subtracted to the remaining number.
D. A divisibility rule in which the unit place digit is multiplied by 5, the multiple is added to the remaining number.
E. A divisibility rule in which the given number is broken into two parts of three digits number and smaller one is subtracted from the greater one.

All the above methods are bit complicated and require long calculation, especially for the numbers having many numbers of digits.

Few other methods, for testing, whether a given number is exactly divisible by 13,23 or the same is not divisible by 13, 23 are –

A. A divisibility rule in which the unit place digit is multiplied by 4,7 respectively for testing the numbers by 13,23 and the multiple is added to the remaining number.

The author proposes a much simpler method than the above ones, which is quicker, faster and can be evaluated orally without using pen and paper. During testing a number by 23, some calculations can be done orally with little difficulty. A common man, who knows simple addition, subtraction, multiplication, division can use these methods. The method is applicable to a number having any number of digits.

2. Test for divisibility of a number by 7

2.1 Rule-1

In the process of testing a number for divisibility by 7, during calculation, if a digit or a number comes , which is equal or more than 7, the digit or the number is made 0 or lesser than 7 by subtracting 7 or 14 (Multiple of 7) from that digit or the number. This is also applicable to the digits of the number to be tested.

The process is started with the digit placed at the highest place value or the left most digit. If the digit is more than 7, it is made less than 7 by subtracting 7 from that digit. The digit placed at the highest place value, which is now less than 7 is multiplied by 2 and the product is added to the digit placed at the third highest place value, after leaving the digit placed at 2nd highest place value. The sum so obtained is multiplied by 2 and added to the digit placed at fifth highest place value. This process is continued till unit place digit or the tens place digit is considered. During calculation, if a digit or a number comes which is more than 7, the same is made less than 7 with the help of Rule 1. This

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way a digit less than 7 or 0 is obtained which is kept at the place where the operation finishes. The same process is started with the digit placed at 2nd highest place value and again a digit less than 7 or 0 is obtained which is kept where the operation finishes. Thus, a two-digit number or 0 is obtained. If this two-digit number is in the table of 7 or 00, the given number is exactly divisible by 7 otherwise the same is not exactly divisible by 7.

2.2 Method Description

The process can better be understood by assuming a number consisting of a, b, c, d, e, f where f is at unit place, e is at tens place and so on. The digit placed at the highest place value (a) is multiplied by 2 and added to the digit 'c'. Mathematically, it can be written as, a x 2 + c = say k, further k x 2 + e = say m. As this process finishes at 'e' which is at tens place of the given number, it is kept at tens place.

The same operation is done with the digit placed at the second highest place value b and the following calculations are made- bx2 +d = say p, p x 2 + f = say n. As this process finishes at f which is at unit place of the given number, the digit n is kept at unit place. This way a two-digit number mn is obtained.

If this two-digit number mn is in the table of 7 or 00, the assumed number is exactly divisible by 7 otherwise the same is not exactly divisible by 7. During calculation, the digit or the number is made less than 7, by Rule 1, as and when required.

2.3 Example

Test whether 123459 is exactly divisible by 7 leaving zero remainder.

Here the digit placed at highest place value is 1, hence the process is started with 1.

1 x 2 + 3 = 5,5 X 2 + 5 = 15, 15 >7 hence 15-14 =1, (By Rule 1)

As the operation finishes at tens place of the given number, 1 is kept at tens place. The digit placed at 2nd highest place value is 2, hence the same process is started with 2

2x2 + 4 = 8, 8 > 7, 8-7 =1 (By Rule 1)

1x2 + 2(9 > 7, 9-7 =2) = 4 (By Rule 1)

As the operation finishes at unit place of the given number, 4 is kept at unit place.

A two-digit number 14 is obtained which is in the table of 7, hence the given number 123459 is exactly divisible by 7, leaving zero remainder.

2.4 Corollary

i. Examining from the above method, it can easily be verified that all six digits and multiple of six digits numbers, having same digit at all places are exactly divisible by7.For example 888888,666666666666, are exactly divisible by 7, because after doing above operation unit place and tens place digits come 0. Numbers other than six digits or multiple of six digits, having same digit at all places, except 7, are not exactly divisible by 7. A six-digit number having 1 and 8 or 2 and 9 as its digit, the number is exactly divisible by 7. For Ex – 118181, 992922 are divisibly by 7 because if the digits are made less than 7, the number becomes 111111, 222222 respectively.

ii. Numbers having same digit at first six digits and other same digit at next six digits and so on are exactly divisible by 7. For example – 444444999999 is exactly divisible by 7.

iii. Four-digit numbers having same digit at unit place and thousands place and having 0 at tens place and hundreds place, are exactly divisible by 7. For example, 6006 is exactly divisible by7. 1008,8001,9002,2009 are exactly divisible by 7 because, when 7 is subtracted from the digits having more than 7, from the numbers, the numbers become 1001,1001,2002,2002, which is exactly divisible by 7. Other such numbers having different digits at unit place and thousands place is not exactly divisible by 7.

3. Test for divisibility of a number by 13

3.1 Rule-2

In the process of testing a number for divisibility by 13 ,during calculation, if a number comes ,which is equal or more than 13 , the number is made 0 or less than 13 by subtracting 13,26 , 39 (the multiples of 13).

The process is started with the digit placed at unit place. The digit placed at unit place is multiplied by 3 and the product is added to the digit placed at hundreds place. After making the number less than 13 by Rule 2, the sum is multiplied by 3 and added to the digit placed at ten thousands place. This process is continued till the digit

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placed at the highest place value or the digit placed at second highest place value is considered.

This way 0 or a number less than 13 is obtained which is kept at the place where the operation finishes. The same process is started with tens place digit and 0 or a number less than 13 is obtained which is kept where the operation finishes.

If the operation finishes at the digit placed at the highest place value, the number 0 or less than 13 is kept at tens place. If the operation finishes at the digit placed at the 2nd highest place value, the number 0 or less than 13 is kept at unit place.

3.2 Method Description

The process can better be understood by assuming a number consisting of a, b, c, d, e, f, where f is at unit place, e is at tens place and so on. The unit place digit f is multiplied by 3 and added to the digit d. Mathematically it can be represented as f x 3 + d = say k, further k x 3 + b = say n .The value of n may be 0 or a number less than 13. As this process finishes at b which is at the digit placed at 2nd highest place value of the assumed number, the digit less than 13 or 0 is kept at unit place. The same operation is done with the digit e, placed at tens place and the following calculations are made-

ex3 + c = say p, p x 3 + a = say m.

As this process finishes at the digit placed at the highest place value, m or 0 is kept at tens place.

This way a two-digit number mn is obtained. If this two-digit number mn is exactly divisible by 13, the assumed number is exactly divisible by 13, otherwise the same is not exactly divisible by 13. During calculation the number is made less than 13 with the help of Rule 2, as and when required.

If m and n are two-digit numbers, the tens place of n is added to the unit place of m and a three digit number is obtained which can be reduced to a two digit number by the method explained above. If n is two-digit number and m is one-digit number, the tens place digit of n is added to the digit at m and a two digit or a three-digit number is obtained. If m is a two-digit number and n is one-digit number a three-digit number is obtained which can be reduced to a two digit number by the method explained above.

3.3 Example

Test weather 162357 is exactly divisible by 13 with zero remainder.

Here unit place digit is 7, hence the process is started with 7

7 x 3 + 3 = 24, 24 > 13, 24 -13 = 11 (By Rule 2)

11 X 3 + 6 = 39, 39 >13 hence 39 -39 = 0, (By Rule 2)

As the operation finishes at the digit placed at 2nd highest place value of the given number, 0 is kept at unit place.

Now tens place digit is 5, hence the same process is started with 5

5x3 + 2 = 17, 17 > 13, 17- 13 = 4 (By Rule 2)

4x3 + 1 = 13, 13 – 13 = 0 (By Rule 2)

As the operation finishes at the digit placed at the highest place value of the given number, 0 is kept at tens place.

A two-digit number 00 is obtained hence the given number 162357 is exactly divisible by 13 leaving zero remainder.

3.4 Corollary

i. Examining from the above method, it can easily be verified that all six digits and multiple of six digits number, having same digit at all places are exactly divisible by 13,for example 888888,666666666666, are exactly divisible by 13, because after above operation unit place and tens place digits come 0 . Numbers other than six digits or multiple of six digits, having same digit at all places, are not exactly divisible by 13.

ii. Numbers having same digit at first six digits and other same digit at next six digits and so on are exactly divisible by 13. For example, 444444999999 is exactly divisible by 13.

iii. A four-digit number having same digit at unit place and thousand place and having 0 at tens place and hundredths place is exactly divisible by 13. For example, 6006 is exactly divisible by 13. Such numbers having different digits at unit place and thousands place is not exactly divisible by 13.

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4. Test for divisibility of a number by 23

4.1 Rule 3

In the process of testing a number for divisibility by 23 ,during calculation, if a number comes ,which is equal or more than 23 , the number is made 0 or less than 23 by subtracting 23, 46, 69 (the multiples of 23).

4.2 Method Description

The method for testing whether a number is exactly divisible by 23, is same as the method explained for the number 13 except that, in place of Rule-2, Rule-3 is applied.

4.3 Example

Test weather 162357 is exactly divisible by 23 with zero remainder.

Here unit place digit is 7, hence the process is started with 7

7 x 3 + 3 = 24, 24 > 23, 24 -23 = 1, 1 X 3 + 6 = 9

As the operation finishes at the digit placed at second highest place value of the given number, 9 is kept at unit place. Now tens place digit is 5, hence the same process is started with 5

5x3 + 2 = 17, 17 x 3 + 1 = 52, 52 > 23, 52 – 46 = 6

As the operation finishes at the digit placed at the highest place value of the given number, 6 is kept at tens place. This way a number 69 is obtained which is exactly divisible by 23 and hence the number 162357 is exactly divisible by 23, leaving zero remainder.

If m and n are two-digit numbers the tens place of n is added to the unit place of m and a three digit number is obtained which can be reduced to a two digit number by the method explained above. If n is two-digit number and m is one-digit number, the tens place digit of n is added to the digit at m and a two digit or a three-digit number is obtained. If m is a two-digit number and n is one-digit number a three-digit number is obtained which can be reduced to a two-digit number by the method explained above.

5 Conclusions

Test for divisibility of a number by a single digit number is known to all and the same is in use since long time but divisibility of a number by 7 is not in use because, the methods developed till date is not practical. The method suggested by the author will be very much helpful, especially to the students. Test for divisibility of a number by 13 is frequently required by the students but till date no suitable method is available. The method suggested above may fulfill this requirement.

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